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INTERVAL-VALUED INTUITIONISTIC FUZZY IDEAL ON SEMI RINGS

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Abstract.

In this paper, the concepts of interval-valued intuitionistic fuzzy ideal on semi-rings are introduced and investigated some of their properties.

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Keywords:

Fuzzy Set, Interval-valued Intuitionistic fuzzy ideal, semi-rings

AMS Mathematics Subject Classification (2010):

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [15], several researches were worked on the generalizations of the notion of fuzzy sets. Interval-valued fuzzy sets provides with a more flexible mathematical framework to deal effectively with imperfect and imprecise information. Atanassov [2,4] introduced the concept of intuitionistic fuzzy sets and the interval-valued intuitionistic fuzzy, as a generation of the notion of fuzzy sets. Interval-valued intuitionistic fuzzy subgroups are discussed [1]. Fuzzy sets and intuitionistic fuzzy sets are widely used in various algebraic systems and other other fields [5, 10, 12, 13, 14].

A. Rosenfeld [11] is the father of fuzzy abstract algebra. He first studied the notion of fuzzy subgroup in 1971. After that in 1979, N. Kuroki [9] introduced the concept of fuzzy semi group. In 1993, J. Ahsan, K. Saifullah and M. Farid Khan [3] introduced the notion of fuzzy semiring. In 1994, T.K. Dutta and B.K. Biswas [6] characterized fuzzy prime ideals of a semiring. K.H. Kim and J. G. Lee [7] studied the intuitionistic fuzzification of the concept of several ideals in a semi groups and investigate some properties of such ideals. K. H. Kim [8] introduced the notion of intuitionistic Q -fuzzy semi primality in a semigroup and investigate some properties of intuitionistic Q-fuzzification of the concept of several ideals. In this paper, the concepts of interval-valued intuitionistic fuzzy ideal on semi-rings are introduced and investigated some of their properties.

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2. PRELIMINARIES

Definition 2.1. A non-empty set S together with two binary operation $+$ and $.$ is said to be a semiring. if

i) $(S,+)$ is a commutative semigroup,
ii) $(S,.)$ is a semigroup,
iii) $a(b + c) = ab + ac$ and$(a + b)c = ac + bc\ \forall\ a,b,c \in S$.

Let $(S,+,.)$ be a semiring. If there exists an element $0_s \in S$ such that $a + 0_s = a = 0_s + a$ and $a.0_s = 0_s = 0_s.a$ for all $a \in S$; then 0_s is called the zero element of S. If there exists an element $1_s \in S$ such that $a.1_s = a = 1_s.a$ for all $a \in S$, then $1_s$is called the identity element of S.

Note 2.1 A semiring may or may not have a zero and an identity element.

We say that a semiring$S$ has a zero.if there exists an element $0 \in S$ such that $0x = x0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$.

Definition 2.2

An interval number on [0,1], denoted by $\hat{a}$, is defined as the closed subinterval of [0,1], where $\hat{a} = [a^-, a^+]$ satisfying $0 \leq a^- \leq a^+ \leq 1$.

For any two interval numbers $\hat{a} = [a^-, a^+]$ and $\hat{b} = [b^-, b^+]$, we define:

i) $\hat{a} \leq \hat{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$.
ii) $\hat{a} = \hat{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$
iii) $\hat{a} < \hat{b}$ if and only if $a^- \neq b^-$ and $\hat{a} \leq \hat{b}$

Definition 2.3

Let $X \neq \phi$ be a set and $A \subseteq X$. Then the interval-valued intuitionistic characteristic function $\chi_A = \left(\tilde{\chi}_{M_A}(x), \tilde{\chi}_{N_A}(x)\right)$ of A is an interval-valued intuitionistic fuzzy subset of X, defined as follows:

$$\hat{\chi}_{M_A}(x) = \begin{cases} \hat{1} & when\ x \in A \\ \hat{0} & when\ x \notin A \end{cases} \text{ and } \hat{\chi}_{N_A}(x) = \begin{cases} \hat{0} & when\ x \in A \\ \hat{1} & when\ x \notin A \end{cases}$$

Definition 2.4

Let $A = (\widehat{M}_A, \widehat{N}_A)$ and $B = (\widehat{M}_B, \widehat{N}_B)$ be two interval-valued intuitionistic fuzzy subsets of a non-empty set X. Then A is said to be subset of B, denoted by $A \subseteq B$, if $\widehat{M}_A(x) \leq \widehat{M}_B(x)$ and $\widehat{N}_A(x) \geq \widehat{N}_B(x)$. (i.e) $M_A^-(x) \leq M_B^-(x)$; $M_A^+(x) \leq M_B^+(x)$; $N_A^-(x) \geq N_B^-(x)$; $N_A^+(x) \geq N_B^+(x)$ for all $x \in X$.

Definition 2.5

Let $A = (\widehat{M}_A, \widehat{N}_A)$ be an interval-valued intuitionistic fuzzy subsets of a non-empty set X and $[\alpha,\beta] \in D[0,1]$. Then the level subset of $A = (\widehat{M}_A, \widehat{N}_A)$, denoted by $\overline{U}(\widehat{M}_A, \widehat{N}_A, [\alpha,\beta])$ is defined as:

$\overline{U}(\widehat{M}_A, \widehat{N}_A, [\alpha,\beta]) = \{\ (x,y)\ :\ \widehat{M}_A(x) \geq [\alpha,\beta],\ \ \widehat{N}_A(x) \leq [\alpha,\beta]\}$

Proposition 2.1
If $[\alpha_1, \beta_1]$ and $[\alpha_2, \beta_2]$ be two interval-valued intuitionistic fuzzy number such that $[\alpha_1, \beta_1] > [\alpha_2, \beta_2]$, then $\overline{U}(\widehat{M}, \widehat{N}, [\alpha_1, \beta_1]) \subseteq \overline{U}(\widehat{M}, \widehat{N}, [\alpha_2, \beta_2])$.

Proof:
Let $[\alpha_1, \beta_1]$ and $[\alpha_2, \beta_2]$ be two interval numbers such that $[\alpha_1, \beta_1] > [\alpha_2, \beta_2]$.
Then $[\alpha_1, \beta_1] \geq [\alpha_2, \beta_2]$ and $[\alpha_1, \beta_1] \neq [\alpha_2, \beta_2]$.
For any $x \in \overline{U}(\widehat{M}, \widehat{N}, [\alpha_1, \beta_1]) \Longrightarrow \widehat{M}(x) \geq [\alpha_1, \beta_1]$ and $\widehat{N}(x) \leq [\alpha_1, \beta_1]$
Then $\widehat{M}(x) \geq [\alpha_1, \beta_1] > [\alpha_2, \beta_2]$ and $\widehat{N}(x) \leq [\alpha_2, \beta_2] < [\alpha_1, \beta_1]$
Therefore, $\widehat{M}(x) > [\alpha_2, \beta_2]$ and $\widehat{N}(x) < [\alpha_2, \beta_2]$
$\therefore\ x \in \overline{U}(\widehat{M}, \widehat{N}, [\alpha_2, \beta_2])$
Thus $\overline{U}(\widehat{M}, \widehat{N}, [\alpha_1, \beta_1]) \subseteq \overline{U}(\widehat{M}, \widehat{N}, [\alpha_2, \beta_2])$

Definition 2.6
The interval $Min-norm$ is a function $Min^i: D[0,1] \to D[0,1] \to D[0,1]$ defined by $Min^i(\hat{a}, \hat{b}) = [\min(a^-, b^-), \min(a^+, b^+)]$ for all $\hat{a}, \hat{b} \in D[0,1]$, where $\hat{a} = [a^-, a^+]$ and $\hat{b} = [b^-, b^+]$.
Definition
The interval $Max-norm$ is a function $Max^i: D[0,1] \to D[0,1] \to D[0,1]$ defined by $Max^i(\hat{a}, \hat{b}) = [\max(a^-, b^-), \max(a^+, b^+)]$ for all $\hat{a}, \hat{b} \in D[0,1]$, where $\hat{a} = [a^-, a^+]$ and $\hat{b} = [b^-, b^+]$.

3 INTERVAL-VALUED INTUITIONISTIC FUZZY IDEAL OF A SEMIRNG

Definition 3.1
A non-empty interval-valued intuitionistic fuzzy subset A of a semiring S is said to be interval-valued intuitionistic fuzzy ideal of S if

1. $M_A(x+y) \geq \min^i\{M_A(x), M_A(y)\}$
2. $N_A(x+y) \leq \max^i\{N_A(x), N_A(y)\}$
3. $M_A(xy) \geq \max^i\{M_A(x), M_A(y)\}$
4. $N_A(xy) \leq \min^i\{N_A(x), N_A(y)\}$ for all $x, y \in S$.

Example 3.1
Let $\mathbb{N}_0$ be a semiring of non-negative integers with respect to usual addition and multiplication. Let A be an interval-valued intuitionistic subset of $\mathbb{N}_0$, defined by

$$M_A(x) = f(x) = \begin{cases} [1,1], & if\ x = 0 \\ [0.5,0.6], & if\ x\ is\ non-zero\ even \\ [0.3,0.4], & if\ x\ is\ odd \end{cases}$$

$$N_A(x) = f(x) = \begin{cases} [0,0], & if\ x = 0 \\ [0.3,0.4], & if\ x\ is\ non-zero\ even \\ [0.5,0.6], & if\ x\ is\ odd \end{cases}$$

Then A is an intuitionistic fuzzy ideal of $\mathbb{N}_0$

Remark 3.1 Let A be an interval-valued intuitionistic fuzzy ideal of a semiring S. Then $M_A^-(0_S) \geq M_A^-(x)$; $M_A^+(0_S) \geq M_A^+(x)$ and $N_A^-(0_S) \leq N_A^-(x)$; $N_A^+(0_S) \leq N_A^+(x)$ for all $x \in S$.

Theorem 3.1
Let S be a semiring and A be a subset of S. Then A is an ideal of S if and only if A is an interval-valued intuitionistic fuzzy ideal of S.
Proof. Let A be an ideal of S. Then $0_S \in A$. So $M_{\chi_A}(0_S) = [1,1]$; $N_{\chi_A}(0_S) = [0,0]$ and hence χ_A is non-empty. Now suppose that *x*, *y* ∈ *S*.
Case: 1
Let $Max^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right) = [0,0]$ and $Min^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right) = [1,1]$.
Then $M_{\chi_A}(x) = [0,0]$ and $M_{\chi_A}(y) = [0,0]$, $N_{\chi_A}(x) = [1,1]$ and $N_{\chi_A}(y) = [1,1]$.
So $M_{\chi_A}(xy) \geq [0,0] = Max^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right)$ and $N_{\chi_A}(xy) \leq [1,1] = Min^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right)$

Case: 2
Let $Max^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right) = [1,1]$ and $Min^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right) = [0,0]$.
Then $M_{\chi_A}(x) = [1,1]$ and $M_{\chi_A}(y) = [1,1]$, $N_{\chi_A}(x) = [0,0]$ and $N_{\chi_A}(y) = [0,0]$.
This implies that $x \in A$ or $y \in A$.
Then $xy \in A$. since A is an ideal of S. This shows that $M_{\chi_A}(xy) = [1,1] = Max^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right)$ and $N_{\chi_A}(xy) = [0,0] = Min^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right)$. Now $Max^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right) = [1,1] \Rightarrow$ $Min^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right) = [0,0]$ or $[1,1]$. $Min^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right) = [0,0] \Rightarrow$ $Max^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right) = [1,1]$.
$Min^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right) = [0,0] \Rightarrow M_{\chi_A}(x+y) \geq Min^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right)$ and
$Max^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right) = [1,1] \Rightarrow N_{\chi_A}(x+y) \leq Max^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right)$

$Max^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right) = [1,1]$, $Min^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right) = [0,0] \Rightarrow M_{\chi_A}(x) = [1,1]$ and $M_{\chi_A}(y) = [1,1]$; $N_{\chi_A}(x) = [0,0]$, $N_{\chi_A}(y) = [0,0]$

$$\Rightarrow x \in A \text{ and } y \in A \Rightarrow x + y \in A$$

Since A is an ideal of S $\Longrightarrow \widehat{M}_{\chi_A}(x+y) = \hat{1} = Min^i\left(M_{\chi_A}(x), M_{\chi_A}(y)\right)$ and $\widehat{N}_{\chi_A}(x+y) = \hat{1} = Max^i\left(N_{\chi_A}(x), N_{\chi_A}(y)\right)$. Consequently $\hat{\chi}_A$ is an interval-valued intuitionistic fuzzy ideal on S.
Conversely, let $\hat{\chi}_A$ be an interval-valued intuitionistic fuzzy ideal on S. Then $\hat{\chi}_A$ is non-empty. So $\widehat{M}_{\chi_A}(x) \neq \hat{0}$ for some $s \in S$. This implies that $\widehat{M}_{\chi_A}(s) = \hat{1}$ and $\widehat{N}_{\chi_A}(s) = \hat{0}$ for some $s \in S$. Hence A is non-empty.
Let $x, y \in A$. Then $\widehat{M}_{\chi_A}(x) = \hat{1} = \widehat{M}_{\chi_A}(y), \widehat{N}_{\chi_A}(x) = \hat{0} = \widehat{N}_{\chi_A}(y)$. Now since $\hat{\chi}_A(x)$ is an interval-valued intuitionistic fuzzy ideal of S, we have

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$\widehat{M}_{\chi_A}(x+y) \geq min^i\{\widehat{M}_{\chi_A}(x), \widehat{M}_{\chi_A}(y)\} = min^i\{\hat{1}, \hat{1}\} = \hat{1}$ and $\widehat{N}_{\chi_A}(x+y) \leq max^i\{\widehat{N}_{\chi_A}(x), \widehat{N}_{\chi_A}(y)\} = max^i\{\hat{0}, \hat{0}\} = \hat{0}$. So $\widehat{M}_{\chi_A}(x+y) \geq \hat{1}$; $\widehat{N}_{\chi_A}(x+y) \leq \hat{0}$. Also $\widehat{M}_{\chi_A}(x+y) \leq \hat{1}$ and $\widehat{N}_{\chi_A}(x+y) \geq \hat{0}$ since $\widehat{M}_{\chi_A}(s) \leq 1$; $\widehat{N}_{\chi_A}(s) \geq 0$ for all $s \in S$.
Thus $\widehat{M}_{\chi_A}(x+y) = \hat{1}$ and $\widehat{N}_{\chi_A}(x+y) = \hat{0}$. So we find that $x+y \in A$ Now, let $a \in A$and $s_1 \in S$. Then $\mathcal{M}_{\chi_A}(a) = \hat{1}, \mathcal{N}_{\chi_A}(a) = \hat{0}$. Now since $\hat{\chi}_A(x)$ is an interval-valued intuitionistic fuzzy ideal of S, we have $\mathcal{M}_{\chi_A}(s_1 a) \geq \max^i\left(\mathcal{M}_{\chi_A}(a), \mathcal{M}_{\chi_A}(s_1)\right) = \hat{1}$ and $\mathcal{N}_{\chi_A}(s_1 a) \leq \min^i\left(\mathcal{N}_{\chi_A}(a), \mathcal{N}_{\chi_A}(s_1)\right) = \hat{0}$. So $\mathcal{M}_{\chi_A}(s_1 a) \geq \hat{1}$ and $\mathcal{N}_{\chi_A}(s_1 a) \leq \hat{0}$.
Thus we find that $\mathcal{M}_{\chi_A}(s_1 a) = \hat{1}$ and $\mathcal{N}_{\chi_A}(s_1 a) = \hat{0}$. Consequently $s_1 a \in A$ Similarly, we can show that $as_1 \in A$ Hence $A$is an ideal of S.

Theorem 3.2
A non-empty interval-valued intuitionistic fuzzy subset $A$of a semiring $S$is an interval-valued intuitionistic fuzzy ideal of Sif and only if $\overline{\mathcal{U}}(\mathcal{M}_{\chi_A}, \mathcal{N}_{\chi_A}[\alpha, \beta])$ are ideals of $S$for all $[\alpha, \beta] \in Im\hat{A}$

Proof: First suppose that $\hat{A}$is an interval-valued intuitionistic fuzzy ideal of S. Let $[\alpha, \beta]$ be an arbitrary element in $Im\hat{A}$ Now consider the level subset $\overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$. Since $[\alpha, \beta] \in Im\hat{A}$, we have $\mathcal{M}_A(s_0) = [\alpha, \beta]$ for some $s_0 \in S$. This implies that $s_0 \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$. So, $\overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$ is non-empty. Now take $x, y \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$. Then we have $\mathcal{M}_A(x) \geq [\alpha, \beta]$ and $\mathcal{N}_A(x) \leq [\alpha, \beta]$; $\mathcal{M}_A(y) \geq [\alpha, \beta]$ and $\mathcal{N}_A(y) \leq [\alpha, \beta]$. Since $\hat{A}$is an interval-valued intuitionistic fuzzy ideal on S, we have $\mathcal{M}_A(x+y) \geq \min^i\{\mathcal{M}_A(x), \mathcal{M}_A(y)\} \geq [\alpha, \beta]$. So, we get $x+y \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$. Again let $a \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$ and $s_1 \in S$. Then $\mathcal{M}_A \geq [\alpha, \beta]$ and $\mathcal{N}_A \leq [\alpha, \beta]$. Since $\hat{A}$ is an interval-valued intuitionistic fuzzy ideal of S, we have $\mathcal{M}_A(s_1 a) \geq Max^i\left(\mathcal{M}_A(s_1), \mathcal{M}_A(a)\right) \geq [\alpha, \beta]$. This implies that $s_1 a \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$. Similarly, we can show that $as_1 \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$. Thus $\overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$ is an ideal of S. Since $[\alpha, \beta]$ is arbitrary, it follows $\overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$ is an ideal of $S$for all $[\alpha, \beta] \in ImA$
Conversely, suppose that $\overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$ are ideal of S for all ImA Let $x, y \in S$ and let $\mathcal{M}_A(x) = [\alpha_1, \beta_1]$; $\mathcal{N}_A(x) = [1-\alpha_1, 1-\beta_1]$ and $\mathcal{M}_A(y) = [\alpha_2, \beta_2]$; $\mathcal{M}_A(y) = [1-\alpha_2, 1-\beta_2]$. This shows that $x \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_1, \beta_1])$ and $y \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_2, \beta_2])$. Without loss of generality, we consider $[\alpha_1, \beta_1] > [\alpha_2, \beta_2]$. Then by theorem ??, we have $\overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_1, \beta_1]) \subseteq \overline{\mathcal{U}}(\mathcal{M}\mathcal{N}[\alpha_2, \beta_2])$. So we find that $x, y \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_2, \beta_2])$. Now since $\overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha, \beta])$ are ideals of S for all $[\alpha, \beta] \in ImA$ $\overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_2, \beta_2])$ is an ideal of S. Thus $x, y \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_2, \beta_2])$ implies that $x+y \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_2, \beta_2])$. Therefore $\mathcal{M}_A(x+y) \geq [\alpha_2, \beta_2] = Min^i\{[\alpha_1, \beta_1], [\alpha_2, \beta_2]\} = Min^i\{\mathcal{M}_A(x), \mathcal{M}_A(y)\}$ and $\mathcal{N}_A(x+y) \leq [\alpha_2, \beta_2] = Max^i\{[\alpha_1, \beta_1], [\alpha_2, \beta_2]\} = Max^i\{\mathcal{N}_A(x), \mathcal{N}_A(y)\}$. Now let $s, t \in S$ be such that $\mathcal{M}_A(t) = [\alpha_3, \beta_3]$. Then $t \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_3, \beta_3])$. Therefore $st \in \overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_3, \beta_3])$, since $\overline{\mathcal{U}}(\mathcal{M}_A, \mathcal{N}_A[\alpha_3, \beta_3])$ is an ideal of S. So $\mathcal{M}_A(st) \geq [\alpha_3, \beta_3] = \mathcal{M}_A(t)$ and $\mathcal{N}_A(st) \leq [\alpha_3, \beta_3] = \mathcal{N}_A(t)$. Similarly, if we take $\mathcal{M}_A(s) = [\alpha_3, \beta_3]$, we can prove that $\mathcal{M}_A(st) \geq \mathcal{M}_A(s)$; $\mathcal{N}_A(st) \leq \mathcal{N}_A(s)$. Consequently, $\mathcal{M}_A(st) \geq Max^i\{\mathcal{M}_A(s), \mathcal{M}_A(t)\}$. Hence $A$is an interval-valued intuitionistic fuzzy ideal of S.

Definition 3.2

If A be an interval-valued intuitionistic fuzzy ideal of S, then the ideals $U(M_A, N_A, [\alpha,\beta])$ of S, where $[\alpha,\beta] \in ImA$ are called the level ideals of A

Theorem 3.2

Let S be a semiring and A be an interval-valued intuitionistic fuzzy ideal on S. Then for any $x, y \in S$; $M_A(x) \geq M_A(y)$; $N_A(x) \leq N_A(y)$ whenever $x \in < y >$, the principal ideal generated by y.

Proof: Since S is a semiring with identity, we find that

$< y >= \{\sum_{i=1}^{n} r_i y s_i : r_i, s_i \in S \, and \, n \in N\}$. Now $x \in < y >$ implies that $x = \sum_{i=1}^{n} r_i y s_i$ for some $r_i, s_i \in S$ and $n \in N$. Then $M_A(x) = M_A(\sum_{i=1}^{n} r_i y s_i)$

$= M_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_n y s_n)$

$= M_A((r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}) + r_n y s_n)$

$\geq Min^i \{M_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}), M_A(r_n y s_n)\}$ (Since A is an interval-valued intuitionistic fuzzy ideal of S.

$\geq Min^i \{M_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}), Max^i (M_A(r_n y), M_A(s_n))\}$ (Since A is an interval-valued intuitionistic fuzzy ideal of S.

$\geq Min^i \{M_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}), Max^i \left(Max^i (M_A(r_n), M_A(y)), M_A(s_n)\right)\}$

$\geq Min^i \{M_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}), M_A(y)\}$

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$\geq M_A(y)$.

Thus we get that $M_A(x) \geq M_A(y)$.

$$N_A(x) = N_A\left(\sum_{i=1}^{n} r_i y s_i\right)$$

$= N_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_n y s_n)$

$= N_A((r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}) + r_n y s_n)$

$\geq Max^i \{N_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}), N_A(r_n y s_n)\}$ (Since A is an interval-valued intuitionistic fuzzy ideal of S.

$\leq Max^i \{N_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}), Min^i (N_A(r_n y), N_A(s_n))\}$ (Since A is an interval-valued intuitionistic fuzzy ideal of S.

$\leq Max^i \{N_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}), Min^i \left(Min^i (N_A(r_n), N_A(y)), N_A(s_n)\right)\}$

$\leq Max^i \{N_A(r_1 y s_1 + r_2 y s_2 + \cdots + r_{n-1} y s_{n-1}), N_A(y)\}$

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$\leq \mathcal{N}_A(y)$.

Thus we get that $\mathcal{N}_A(x) \leq \mathcal{N}_A(y)$.

Theorem 3.4

Let I be an ideal of a semiring S and $[\alpha,\beta] \leq [\gamma,\delta] \neq \hat{0}$ be any two interval valued intuitionistic fuzzy numbers on $[0,1]$. Then the interval-valued intuitionistic fuzzy subset A of S defined by

$\mathcal{M}_A(x) = \begin{cases} [\gamma,\delta] & when\ x \in I \\ [\alpha,\beta] & otherwise \end{cases}$ $\mathcal{N}_A(x) = \begin{cases} [1-\gamma, 1-\delta] & when\ x \in I \\ [1-\alpha, 1-\beta] & otherwise \end{cases}$ is an interval-valued intuitionistic fuzzy ideal of S.

Proof: Since I is an ideal of S, we have $0_S \in I$. Then $\mathcal{M}_A(0_S) = [\gamma,\delta] \neq \hat{0}$ and $\mathcal{N}_A(0_S) = [1-\gamma, 1-\delta] \neq \hat{0}$. So $\mathcal{M}_A(x)$ is non-empty.

Now let $x, y \in S$.

Case: 1

Let $Max^i(\mathcal{M}_A(x), \mathcal{M}_A(y)) = [\alpha,\beta]$ and $n^i(\mathcal{N}_A(x), \mathcal{N}_A(y) = [1-\alpha, 1-\beta]$.

Then $\mathcal{M}_A(x) = [\alpha,\beta]$ and $\mathcal{N}_A(x) = [1-\alpha, 1-\beta]$.

$\mathcal{M}_A(y) = [\alpha,\beta] \Rightarrow \mathcal{M}_A(xy) \geq [\alpha,\beta] = Max^i(\mathcal{M}_A(x), \mathcal{M}_A(y)$ and

$\mathcal{N}_A(y) = [1-\alpha, 1-\beta] \Rightarrow \mathcal{N}_A(xy) \leq [[1-\alpha, 1-\beta]$ and $\mathcal{M}_A(x+y) \geq [\alpha,\beta] = Min^i(\mathcal{M}_A(x), \mathcal{M}_A(y))$ and

$\mathcal{N}_A(x+y) \leq [1-\alpha, 1-\beta] = Max^i(\mathcal{M}_A(x), \mathcal{M}_A(y))$.

Case: ii

Let $Max^i(\mathcal{M}_A(x), \mathcal{M}_A(y)) = [\gamma,\delta]$ and $Mn^i(\mathcal{N}_A(x), \mathcal{N}_A(y)) = [1-\gamma, 1-\delta]$.

Then $\mathcal{M}_A(x) = [\gamma,\delta]$; $\mathcal{N}_A(x) = [1-\gamma, 1-\delta]$; or $\mathcal{M}_A(y) = [\gamma,\delta]$; $\mathcal{N}_A(y) = [1-\gamma, 1-\delta]$;

$\Rightarrow x \in I$ or $y \in I \Rightarrow xy \in I$ (since I is an ideal of S)

$\Rightarrow \mathcal{M}_A(xy) = [\gamma,\delta] = Max^i(\mathcal{M}_A(x), \mathcal{M}_A(y))$ and $\mathcal{N}_A(xy) = [1-\gamma, 1-\delta] = Min^i(\mathcal{N}_A(x), \mathcal{N}_A(y))$

Now, $Max^i(\mathcal{M}_A(x), \mathcal{M}_A(y)) = [\gamma,\delta] \Rightarrow Min^i(\mathcal{M}_A(x), \mathcal{M}_A(y)) = [\alpha,\beta]$ or $[\gamma,\delta]$

$Min^i(\mathcal{N}_A(x), \mathcal{N}_A(y)) = [1-\gamma, 1-\delta] \Rightarrow Max^i(\mathcal{N}_A(x), \mathcal{N}_A(y)) = [1-\alpha, 1-\beta]$ or $[1-\gamma, 1-\delta]$

$Min^i(\mathcal{M}_A(x), \mathcal{M}_A(y)) = [\alpha,\beta] \Rightarrow \mathcal{M}_A(x+y) \geq [\alpha,\beta] = Min^i(\mathcal{M}_A(x), \mathcal{M}_A(y))$ and

$Max^i(\mathcal{N}_A(x), \mathcal{N}_A(y)) = [1-\alpha, 1-\beta] \Rightarrow \mathcal{N}_A(x+y) \leq [1-\alpha, 1-\beta] = Max^i(\mathcal{N}_A(x), \mathcal{N}_A(y))$

$Min^i(\mathcal{M}_A(x), \mathcal{M}_A(y)) = [\gamma,\delta] \Rightarrow \mathcal{M}_A(x) = [\gamma,\delta]$ and $\mathcal{M}_A(y) = [\gamma,\delta] \Rightarrow x \in I$ and $y \in I \Rightarrow x+y \in I$

$Max^i(\mathcal{N}_A(x), \mathcal{N}_A(y)) = [1-\gamma, 1-\delta] \Rightarrow \mathcal{N}_A(x) = [1-\gamma, 1-\delta]$ and $\mathcal{N}_A(y) = [1-\gamma, 1-\delta] \Rightarrow x \in I$ and $y \in I \Rightarrow x+y \in I$ (since I is an ideal of S).

$\Rightarrow \mathcal{M}_A(x+y) = [\gamma,\delta] = Min^i(\mathcal{M}_A(x), \mathcal{M}_A(y))$ and $\mathcal{N}_A(x+y) = [1-\gamma, 1-\delta] = Max^i(\mathcal{N}_A(x), \mathcal{N}_A(y))$

$\mathcal{M}_A(xy) \geq Max^i(\mathcal{M}_A(x), \mathcal{M}_A(y))$ and $\mathcal{N}_A(xy) \leq Min^i(\mathcal{N}_A(x), \mathcal{N}_A(y))$

Consequently, $\mathcal{M}_A$ is an interval-valued fuzzy ideal of S.

Theorem 3.5

Let $\mathcal{A}$ be an interval-valued intuitionistic fuzzy ideal of a semiring $\mathcal{S}$. Then the set $\tilde{\mathcal{A}}_0 = (\mathcal{M}_0, \mathcal{N}_0)$ where $\mathcal{M}_0 = \{x \in \mathcal{S}: \mathcal{M}_A(x) = \mathcal{M}_A(0_s)\}$ and $\mathcal{N}_0 = \{x \in \mathcal{S}: \mathcal{N}_A(x) = \mathcal{N}_A(0_s)\}$ is an ideal of $\mathcal{S}$.

Proof:

Since $0_s \in \tilde{\mathcal{A}}_0$ is non-empty. Let $x, y \in \tilde{\mathcal{A}}_0$. Then $\tilde{\mathcal{A}}(x) = \tilde{\mathcal{A}}_0(0_s) = \tilde{\mathcal{A}}(y)$. Now since $\tilde{\mathcal{A}}$ is an interval-valued intuitionistic fuzzy ideal of $\mathcal{S}$, we have $\mathcal{M}_A(x+y) \geq Min^i\left(\mathcal{M}_A(x), \mathcal{M}_A(y)\right) = \mathcal{M}_A(0_s)$ and $\mathcal{N}_A(x+y) \leq Max^i\left(\mathcal{N}_A(x), \mathcal{N}_A(y)\right) = \mathcal{N}_A(0_s)$. Also by remark 1, we have $\mathcal{M}_A(0_s) \geq \mathcal{M}_A(x+y)$; $\mathcal{N}_A(0_s) \geq \mathcal{N}_A(x+y)$. Thus $\mathcal{M}_A(x+y) = \mathcal{M}_A(0_s)$; $\mathcal{N}_A(x+y) = \mathcal{N}_A(0_s)$. So $x+y \in \tilde{\mathcal{A}}_0$.

Let $s \in \mathcal{S}$ and $t \in \tilde{\mathcal{A}}_0$. Then $\mathcal{M}_A(t) = \mathcal{M}_A(0_S)$; $\mathcal{N}_A(t) = \mathcal{N}_A(0_s)$.

Now since $\tilde{\mathcal{A}}$ is an interval-valued intuitionistic fuzzy ideal of $\mathcal{S}$, we have

$\mathcal{M}_A(st) \geq Max^i\left(\mathcal{M}_A(s), \mathcal{M}_A(t)\right) = Max^i\left(\mathcal{M}_A(s), \mathcal{M}_A(0_s)\right) = \mathcal{M}_A(0_S)$ and

$\mathcal{N}_A(st) \leq Min^i\left(\mathcal{N}_A(s), \mathcal{N}_A(t)\right) = Min^i\left(\mathcal{N}_A(s), \mathcal{N}_A(0_s)\right) = \mathcal{N}_A(0_S)$

Also, since $\mathcal{M}_A(0_s) \geq \mathcal{M}_A(st)$; $\mathcal{N}_A(0_s) \leq \mathcal{N}_A(st)$, we have

$\mathcal{M}_A(st) = \mathcal{M}_A(0_s)$; $\mathcal{N}_A(st) = \mathcal{N}_A(0_s)$. Thus $st \in \tilde{\mathcal{A}}_0$. Similarly, we can show that $ts \in \tilde{\mathcal{A}}_0$.

Hence $\tilde{\mathcal{A}}_0$ is an ideal of $\mathcal{S}$.

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